

Substitution for Substructural Theories

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Abstract. Algebraic theories as monoids for a substitution monoidal structure [4] have been considered in substructural settings, such as for linear theories [5] and affine theories [6]. We will recast these constructions noting that they arise from various free symmetric monoidal theories [1], and further generalise them to a theory encompassing linear, affine, relevant, and cartesian structures together with substructural coercions between them. Thereon, we develop a substitution monoidal structure on the corresponding presheaf category and establish that its monoids are equivalent to substructural abstract clones. This generalises the well-known equivalences of monoids for the substitution monoidal structure in the cartesian and linear settings with abstract clones and symmetric operads, respectively.

Following [2, 3], we extend the aforementioned monoidal category to a bicategory that is a model of linear logic and thus induces a cartesian closed bicategory. We similarly consider such bicategories for the various subtheories and extend each subtheory inclusion to a pseudo-adjunction. This yields various free-forgetful adjunctions between categories of theories, such as between Lawvere theories, symmetric operads, and our substructural theories.

References

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